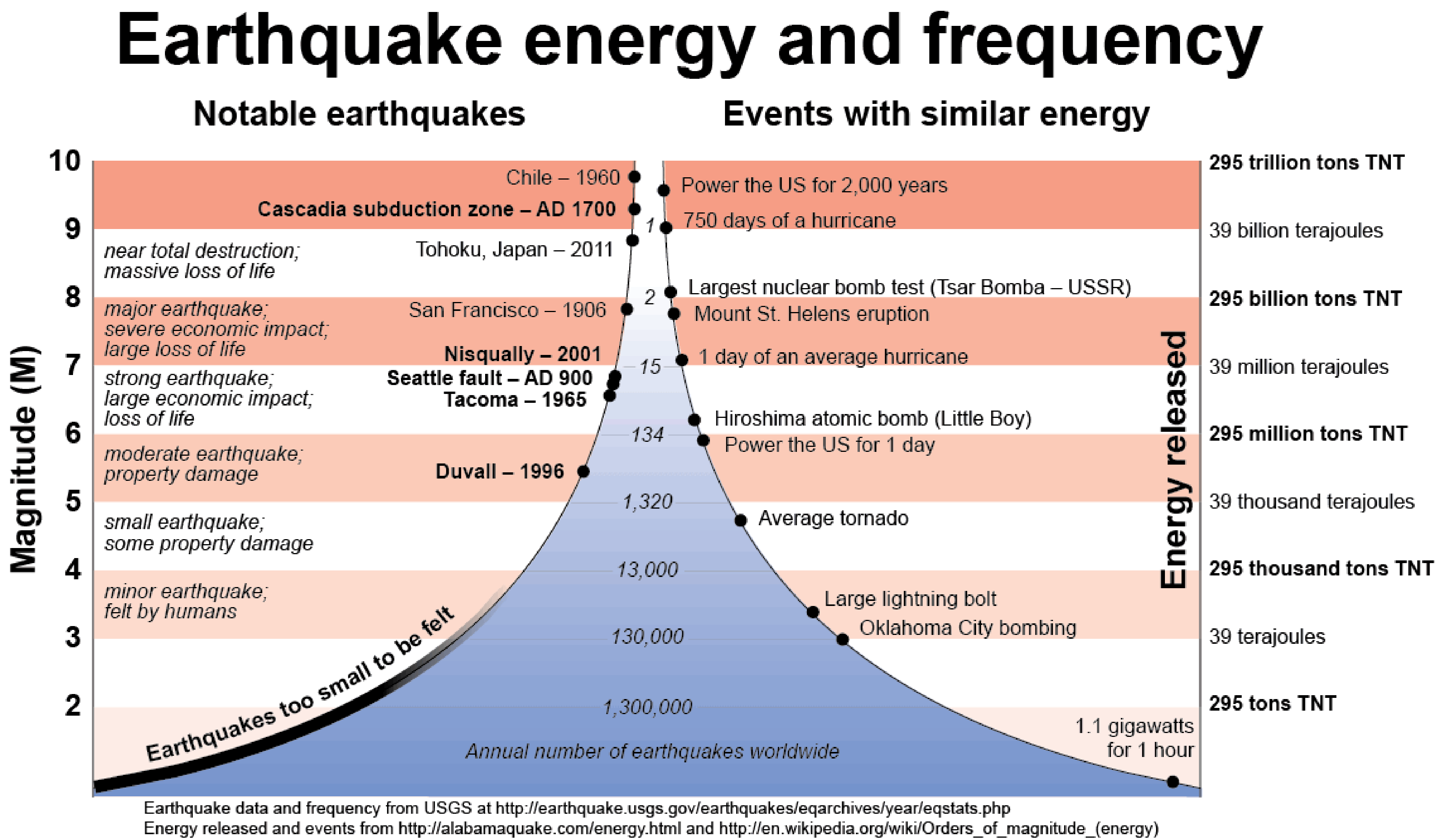
Engineering Seismology and Seismic Hazard – 2019

Lecture 16

Seismicity Analysis

Valerio Poggi
Seismological Research Center (CRS)
National Institute of Oceanography and Applied Geophysics (OGS)

Energy and Occurrence



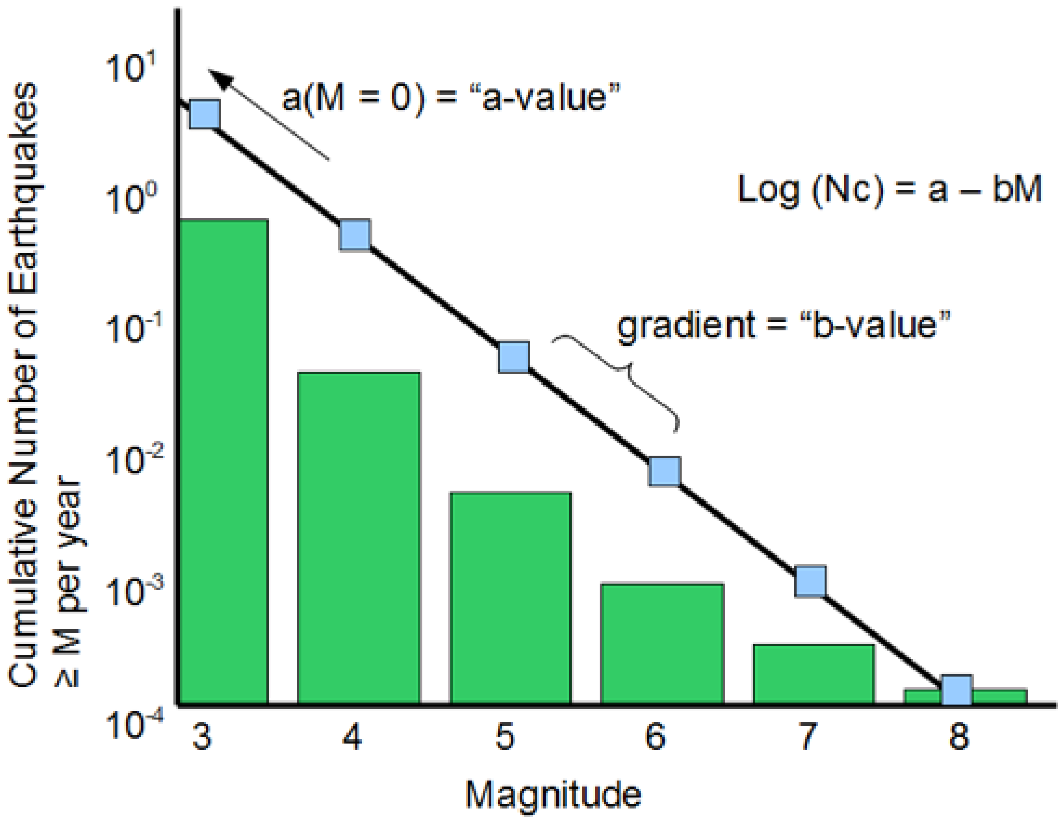
Gutenberg-Richter Law

Guthenberg and Richter observed in 1944 that the **cumulative number of earthquakes** usually scales linearly with magnitude (M_L), according to the law:

$$\log_{10}(N_c) = a - bM_L$$

a = intercept, represents the seismic productivity of the region (at M=0)

b = slope, represents the relative proportion between small and large events



Gutenberg-Richter Law

It is not uncommon a representation of the G-R relation in natural log, which can be equivalently obtained as:

$$N = 10^{a-bM} = e^{\alpha - \beta M}$$

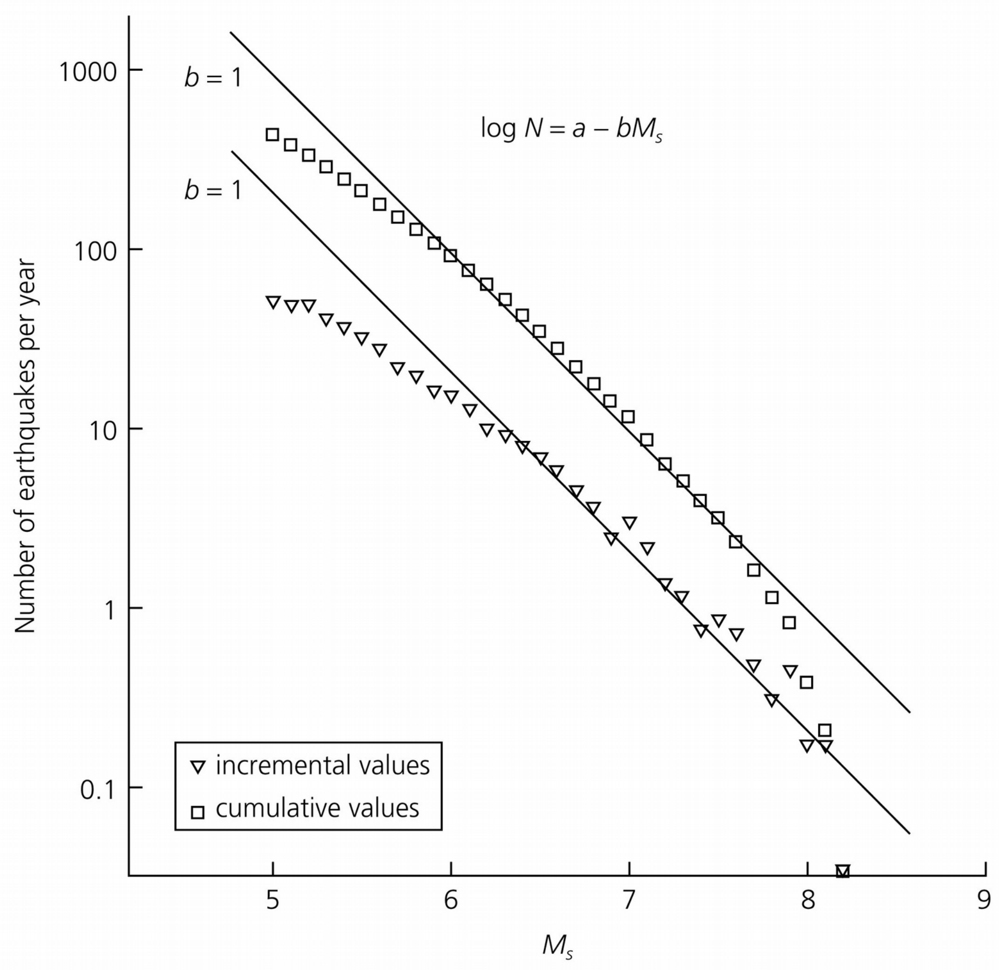
$$\log_e\left(10^{a-bM}\right) = \log_e\left(e^{\alpha-\beta M}\right)$$

$$(a - bM)\log_e(10) = \alpha - \beta M$$

$$\Rightarrow \quad \alpha = 2.303\,a \qquad \beta = 2.303\,b$$

Cumulative vs Incremental

Although the relation has be originally defined for cumulative events, it is sometimes useful its representation in incremental magnitude bins.



Using the G-R relation

Let's do a little extrapolation exercise:

Suppose b=1, there are two M 5.0+ earthquakes per year in the region.

1) Which is the a-value?

$\log(N) = a - bM$
$\log(2) = a - (1)(5)$
$a = 5.30$

2) How often does an M 7.0+ occur?

$\log(N) = 5.30 - (1)(7) = -1.7$
$N = 10^{(-1.7)} = 0.01995$
which is about 2 events every 100 years.

b-value calculation

Different approaches to fit a G-R relation exist:

A) **least square method (LSQ)**

This approach consists in fitting a straight line to N vs M. It works well on incremental occurrences, but is formally incorrect on cumulative, as it would break the assumption of independent samples.

In fact, LSQ assumes the error at each point is Gaussian rather than Poissonian (we will come back to this later).

The method could be disproportionally influenced by large earthquakes.

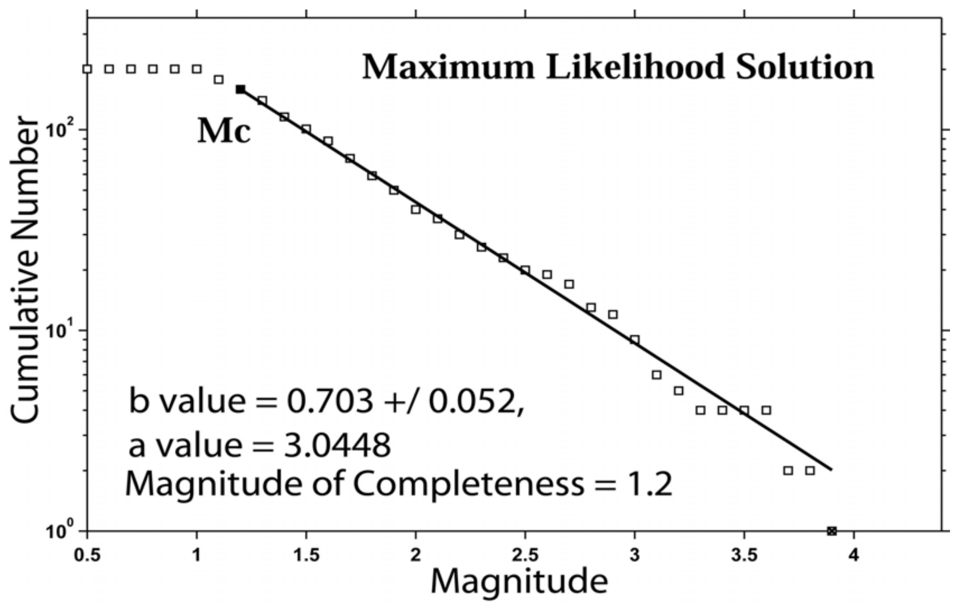
b-value calculation

B) **Maximum likelihood (MLE)** and variants (Aki, Weichert)

It can be applied directly on cumulative samples. MLE weights each earthquake proportionally.

$$b = \frac{\log_{10} e}{\overline{M} - M_{min}}$$

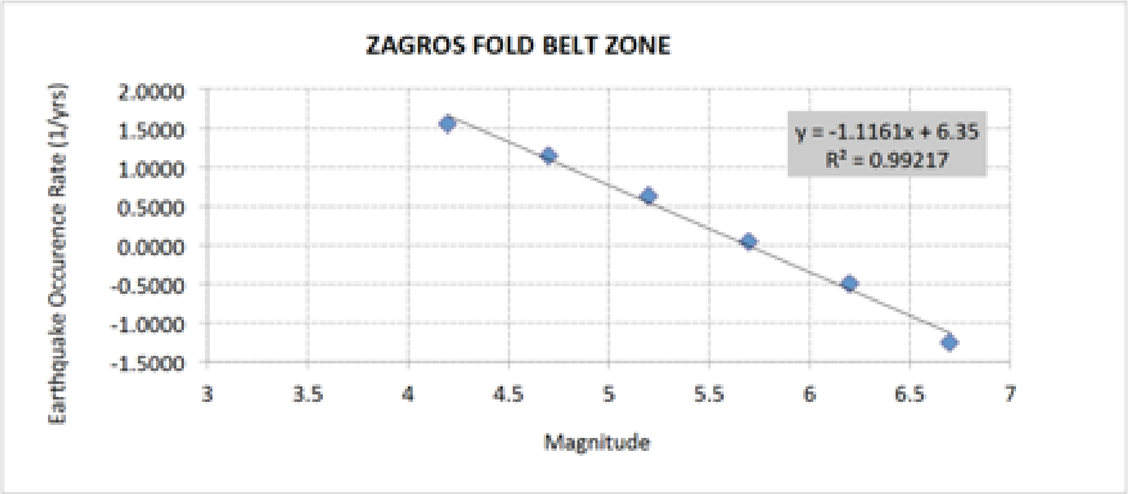
Where Mmin is the smallest earthquake in the catalogue, while Mnot is the average.



Example

Zagros Fold Belt – Recurrence parameters

	4.2	4.7	5.2	5.7	6.2	6.7
Selected rate	21.000	9.903	3.113	0.789	0.254	0.056
Cumulative rate (N)	35.114	14.1145	4.211	1.0986	0.3099	0.056
log(N)	1.5455	1.1497	0.6244	0.0408	-0.5088	-1.2492
Computed rate	45.960	12.715	3.518	0.973	0.269	0.074



A= 6.35
B=1.1161

Typical b-value ranges

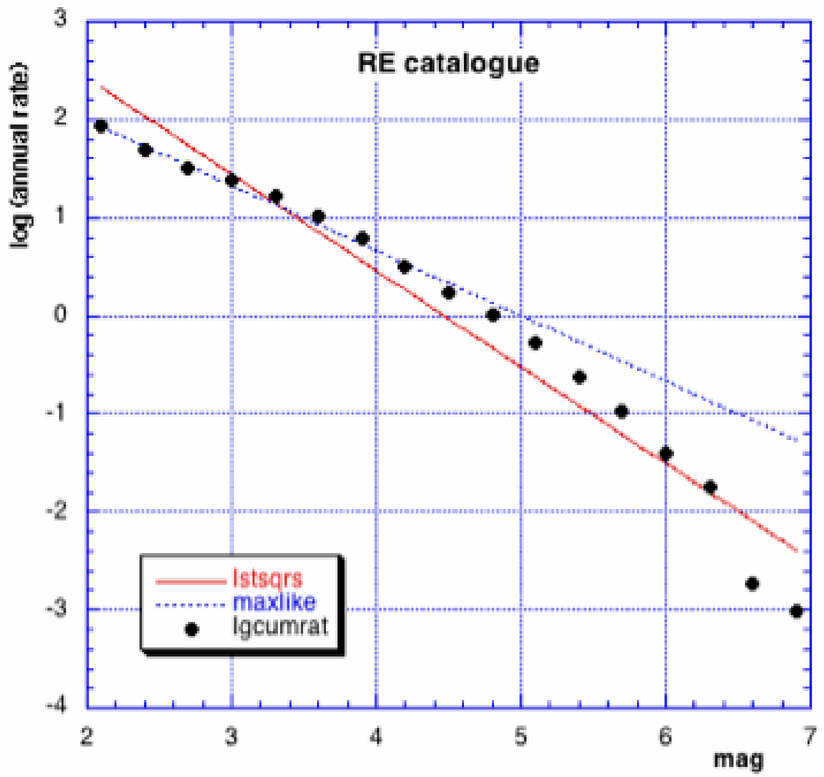
The b-value is often approximated to 1, however there are differences between tectonic regions:

(a) b-value is typically in the range of 0.8-1.1.
(b) 1.5 to 2.0 for volcanic region
(c) 1.0 to 1.5 for oceanic ridge
(d) 0.7 to 1.0 for interplate
(e) 0.5 to 0.7 for subduction interface
(f) $1.0 \le b \le 1.6$ Mogi, global seismicity, b~1.0 for lat. ≥ 40 , whereas b~1.6 for lat. ≤ 40
(g) $0.3 \le b \le 1.8$ Hurtig and Stiller (1984), global seismicity
(h) $0.6 \le b \le 1.5$ Udias and Mezcua (1997), global seismicity
(i) $0.8 \le b \le 1.2$ McNally (1989), global seismicity
(j) $0.5 \le b \le 1.5$ McGarr (1984), mining tremors (South Africa) and tectonic earthquakes
(k) $0.6 \le b \le 1.6$ Monterroso and Kulhanek (2003), Central America seismicity
(l) $0.6 \le b \le 2.6$ Nuannin et al.(2002), mining tremors, Zinkgruvan, Sweden

Common Errors

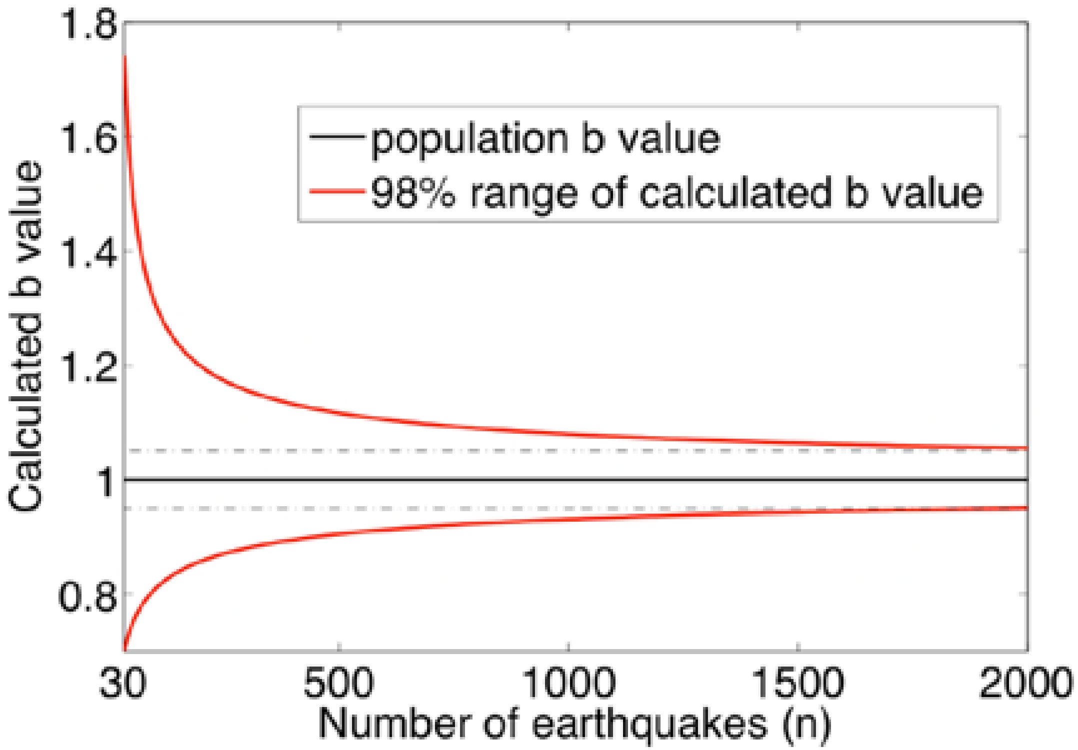
- Dataset is too small
- Using earthquakes smaller than the catalog completeness threshold
- Using data with magnitude errors
- Fitting cumulative data with linear least squares (LSQ) rather than the simple maximum likelihood (MLE) method

Neglecting these source of uncertainty will introduce biases on hazard analysis, and ad-hoc strategies must be implemented (e.g. logic-tree).



Small Dataset

In principle >2000 good quality earthquakes are required for 98% confidence errors < 0.05.
Such amount is usually not available, especially for small and low-seismicity regions.

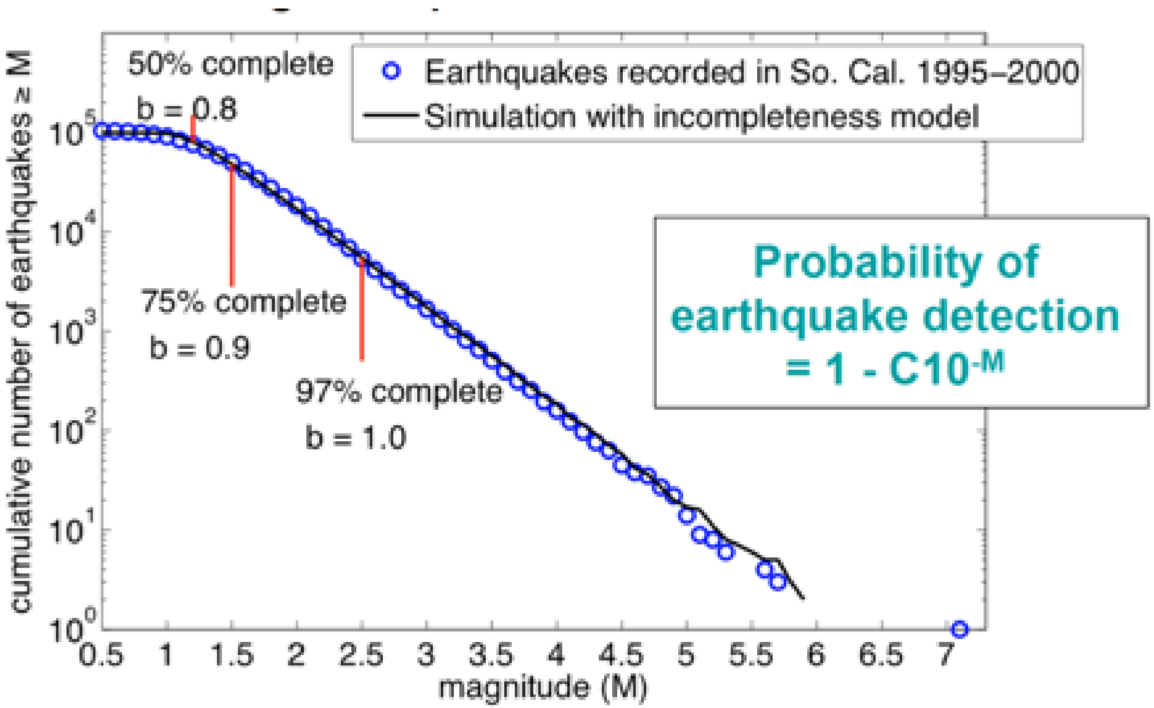


n	*b* range
30	0.7 - 1.74
50	0.5 - 1.49
100	0.86 - 1.20
500	0.91 - 1.12

Completeness Threshold

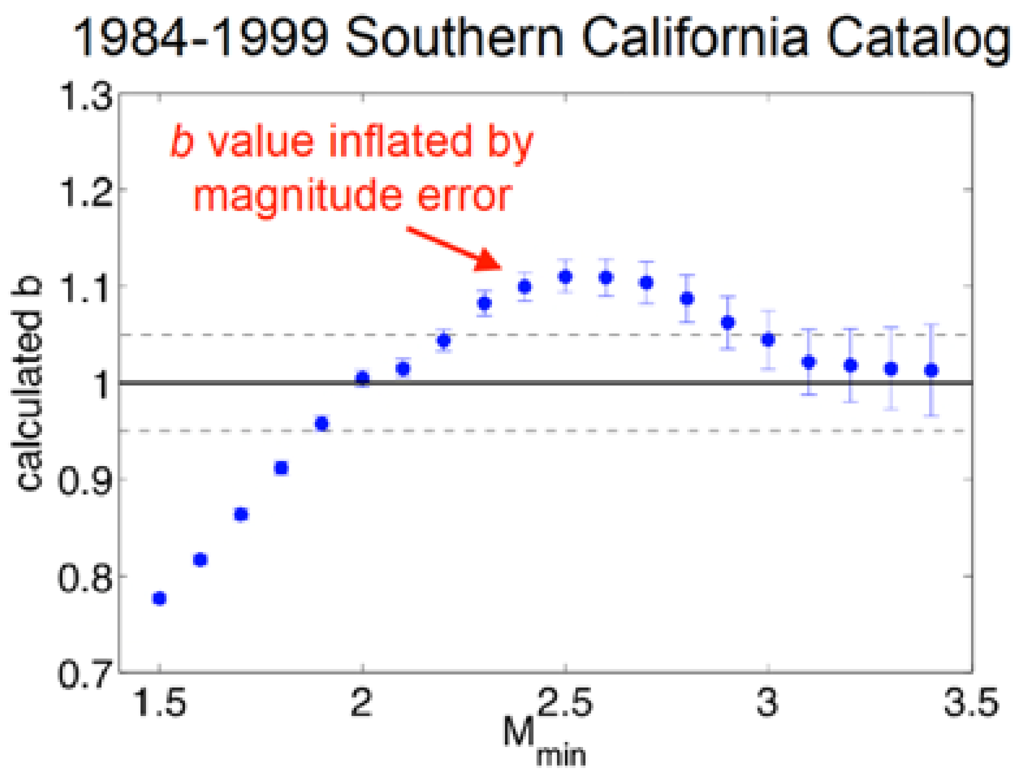
Using earthquakes smaller than the catalog completeness threshold can have a large impact on the result, e.g.:

(1) b value error as small as 0.05 will cause the calculated rate of M ≥ 6.5 earthquakes to be off by 25%,
(2) b value error of 0.1 will cause the M ≥ 6.5 rates to be off by 50%.



Errors on Magnitude

Larger magnitude errors for smaller earthquakes inflate b, while b is best fit at the largest reasonable minimum magnitude.



Unfortunately, magnitude error estimates are rarely available, so quantification of uncertainty is difficult.

M_{MIN} and M_{MAX}

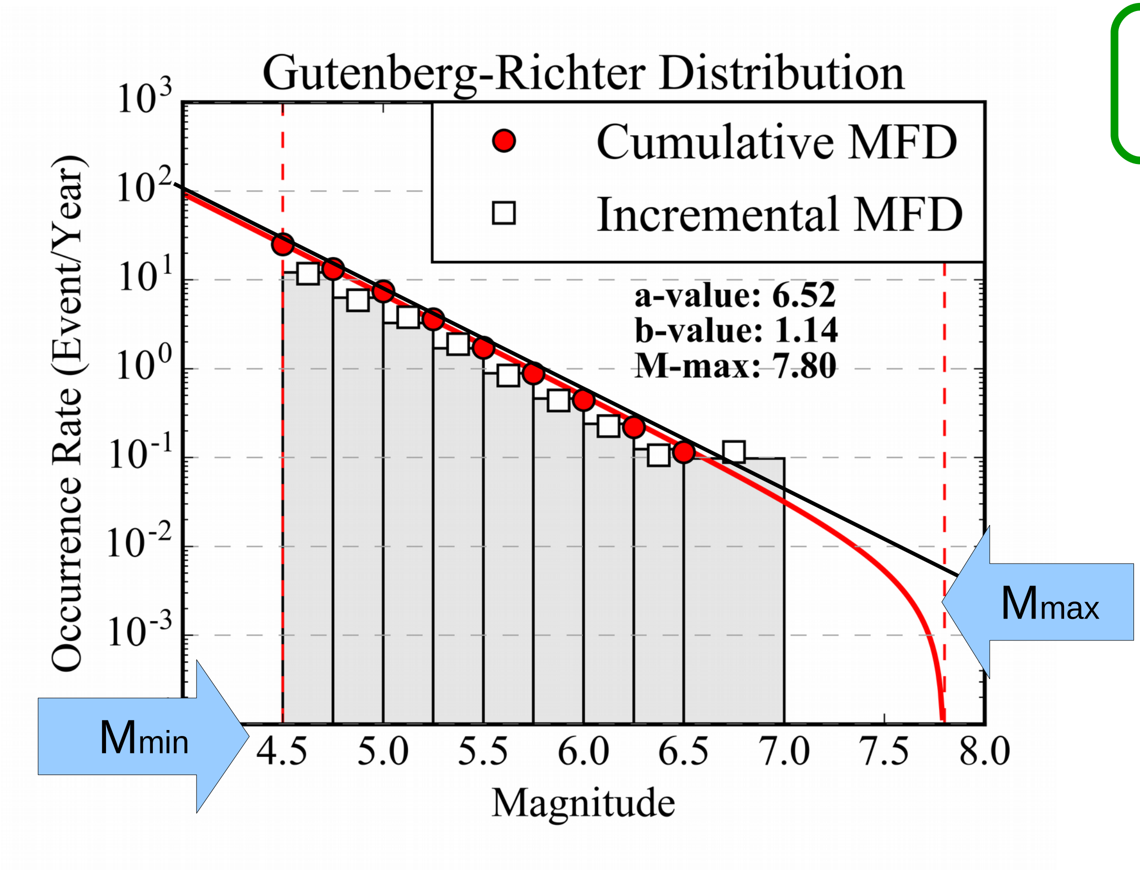
In its original form, G-R relation has no minimum and maximum bounds. This is ineffective as:

1) Too small magnitudes are incomplete and generally not significant to engineering applications (depending on case).

2) It is unrealistic to assume that any large magnitude can be generated, event with very small occurrence. There is a need to define the maximum possible or credible earthquake, however this limits is difficult and controversial to be identified!

Truncated G-R Relation

For these reasons it is more suitable the use of modified G-R relation that accounts for Mmin and Mmax. This is called **bounded or truncated G-R law**:



$$N_c = 10^a 10^{b(M_{max} - M)}$$

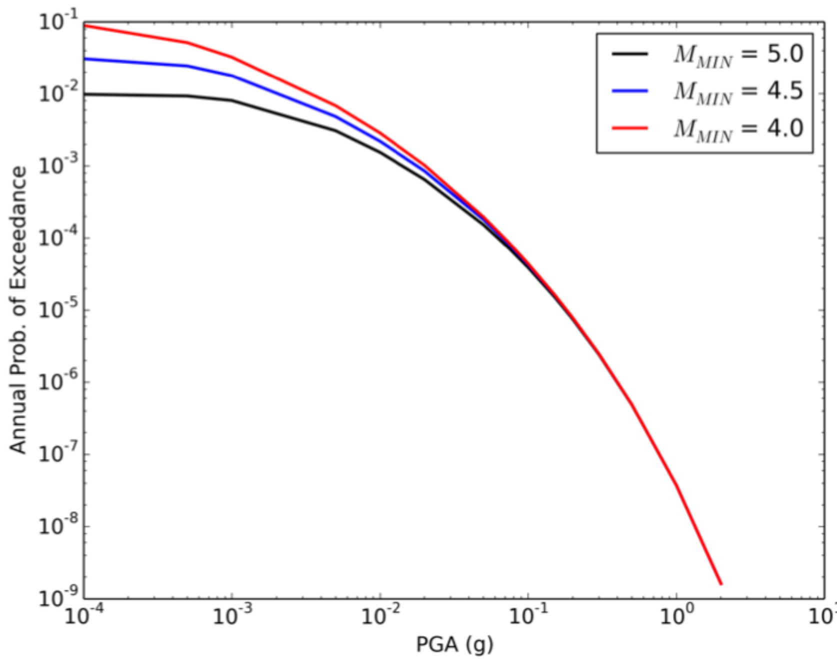
M_{MAX} Estimation

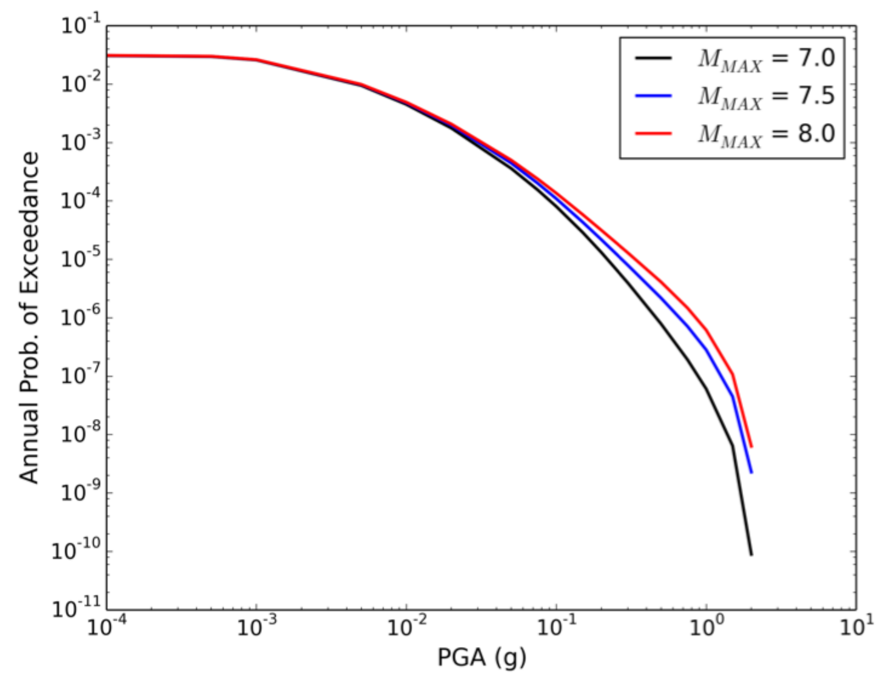
Method	Notes
M_{MAX} = Maximum Observed Magnitude (obs[M_{MAX}])	• Quick & Easy • Usually incorrect – very likely to be an underestimate unless record captures many loading and release cycles
M_{MAX} = obs[MMAX] + ΔM	• Quick, Easy and a little more conservative • Arbitrary and risks underestimating
Inferred from recurrence (i.e. very low probability)	• Quick, Easy and consistent with recurrence model • Not technically a "Maximum Magnitude"
Local geological features	• Physically consistent with the geology • For area sources, geological features not well defined
Maximum Likelihood (Kijko, 2004)	• Stronger statistical basis (can adapt to uncertain recurrence models and parameters) • An underestimate unless several strain cycles observed
Regional/Global Analogues (EPRI, 1994; 2012)	• Robust and consistent with tectonic environment • Very dependent on regionalisation • Large (but probably well-constrained) uncertainties,

Impact on Hazard

1) Decreasing M_{MIN} increases the probability of exceeding smaller ground motions – raising the hazard at higher probabilities. Effect is reduced at longer spectral periods

2) Increasing M_{MAX} increases the probability of exceeding larger ground motions – raising the hazard at lower probabilities. Effect is more pronounced at longer spectral periods.

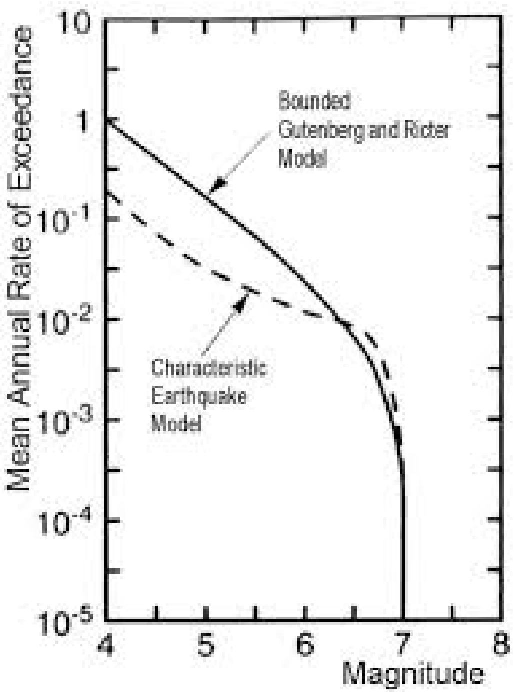




The Characteristic Model

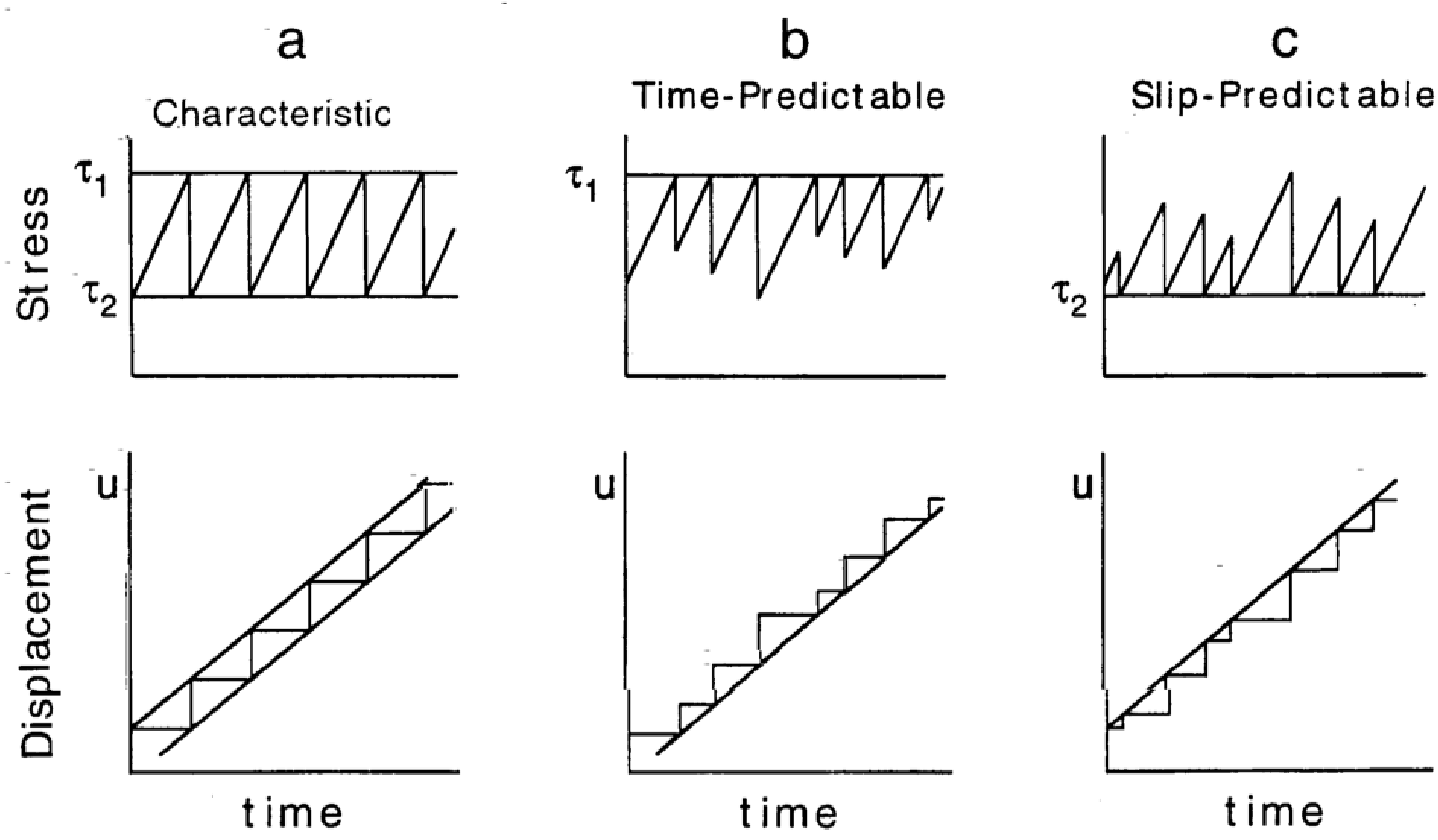
- For large regions with many earthquakes the Gutenberg-Richter model works well
- On individual faults, however, it is common to see repeated events with similar magnitude
- Events of such magnitude may be related with segments (we will revisit these concepts later)
- Often modeled as a Gaussian (or sometimes Dirac) function, centered around a characteristic magnitude.

$$f\left(m|m_{char},\sigma,a,b\right)=\frac{\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{m-m_{char}}{\sigma}\right)^2\right)}{\Phi\left(b\right)-\Phi\left(a\right)}$$



Shimazaki Models

Assuming that loading (tectonic strain) is constant recurrence on individual segments of faults may theoretically correspond to one of two types of behavior: time-predictable (assumed a fixed critical strain level) or slip-predictable (assumes a "base-line" level to which strain returns after a seismic event)

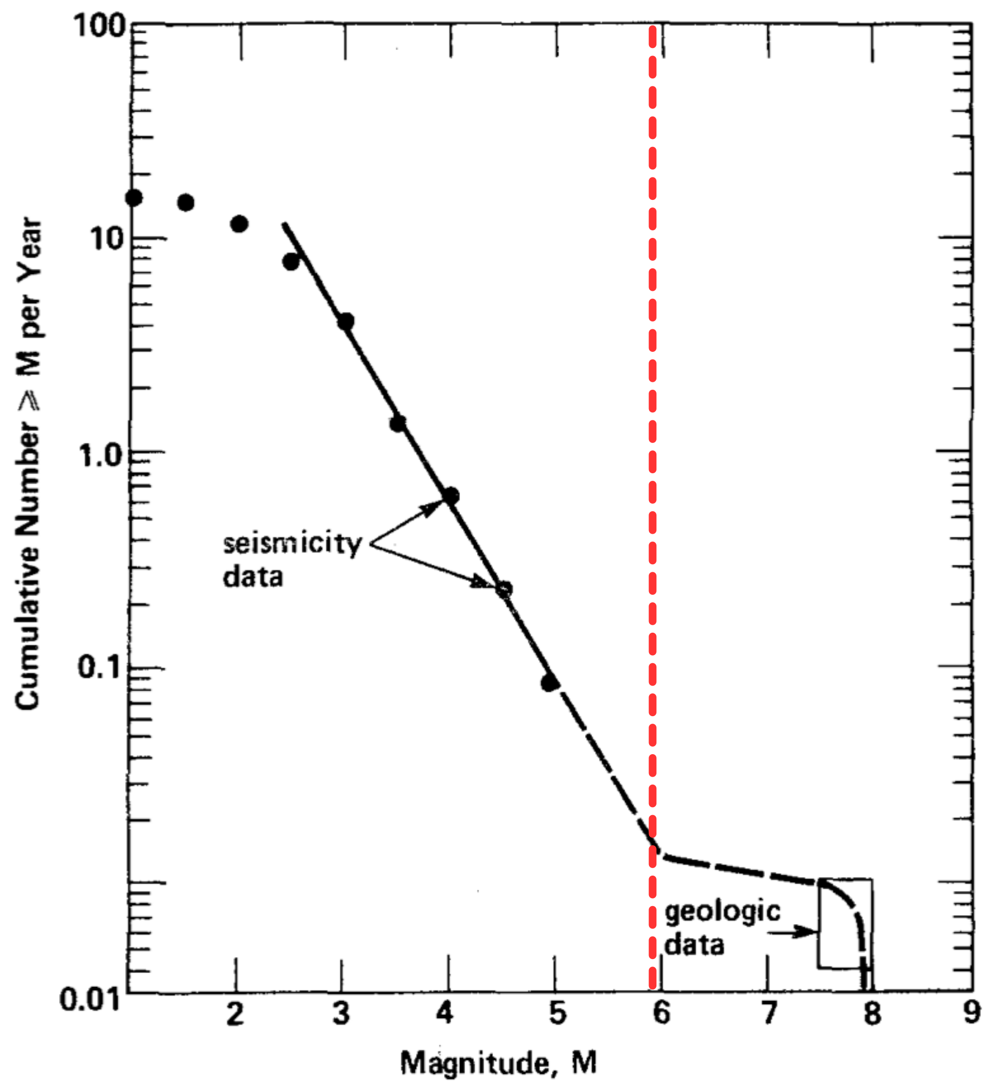


The Hybrid G-R Model

Characteristic model is not always consistent with observations of small to moderate seismicity on faults.

An hybrid model instead distributes small earthquakes exponentially, but gives a higher rate to large events.

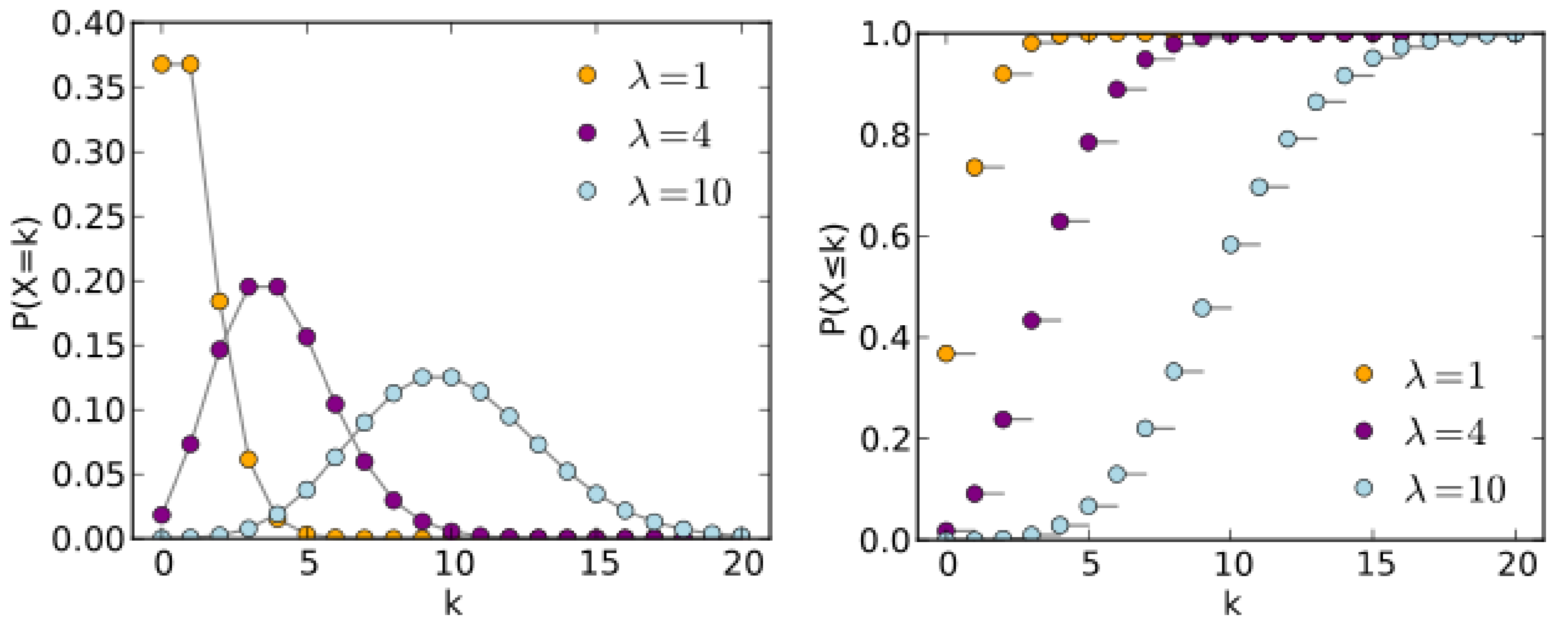
A popular hybrid model is from Youngs and Coppersmith (1985).



Poisson Assumption

A Poisson distribution is a probability distribution that characterizes discrete events occurring independently of one another in time.

A common (although questionable) assumption in probabilistic seismic hazard analysis is that earthquakes occurrence follow a Poisson process for long-term activity rates.



Main Assumptions

A Poisson process requires three assumptions:

Stationarity: The rate of occurrence (λ) is constant (also results in proportionality)

Independence: The number of occurrences in a given interval does not dependent on the number of occurrences in preceding intervals

Non-simultaneity: The probability of simultaneous occurrences is zero

Are these assumptions (always) valid?

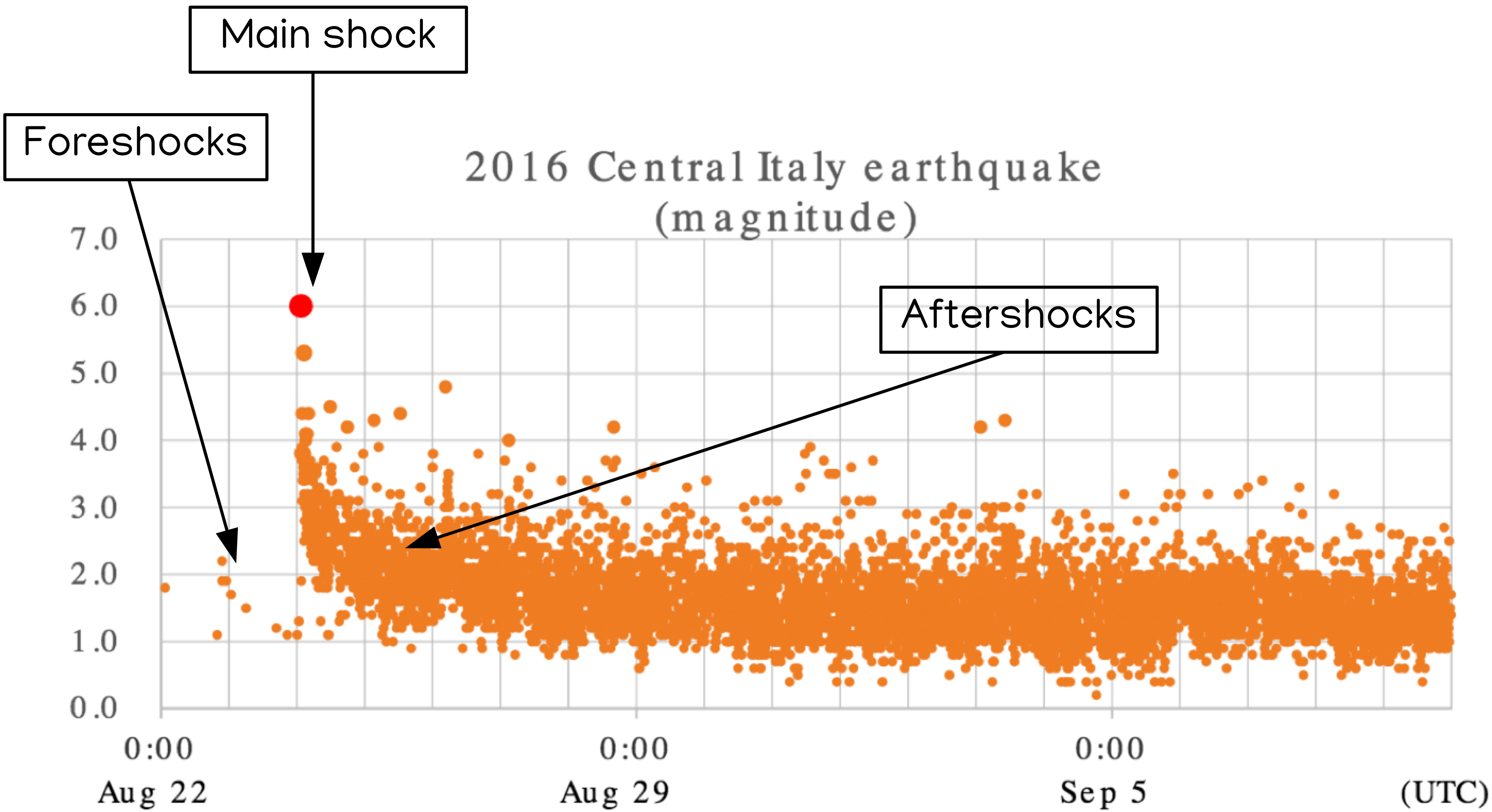
Catalogue Declustering

Decluttering is the process of separating an earthquake catalog into foreshocks, mainshocks, aftershocks or multiplets:

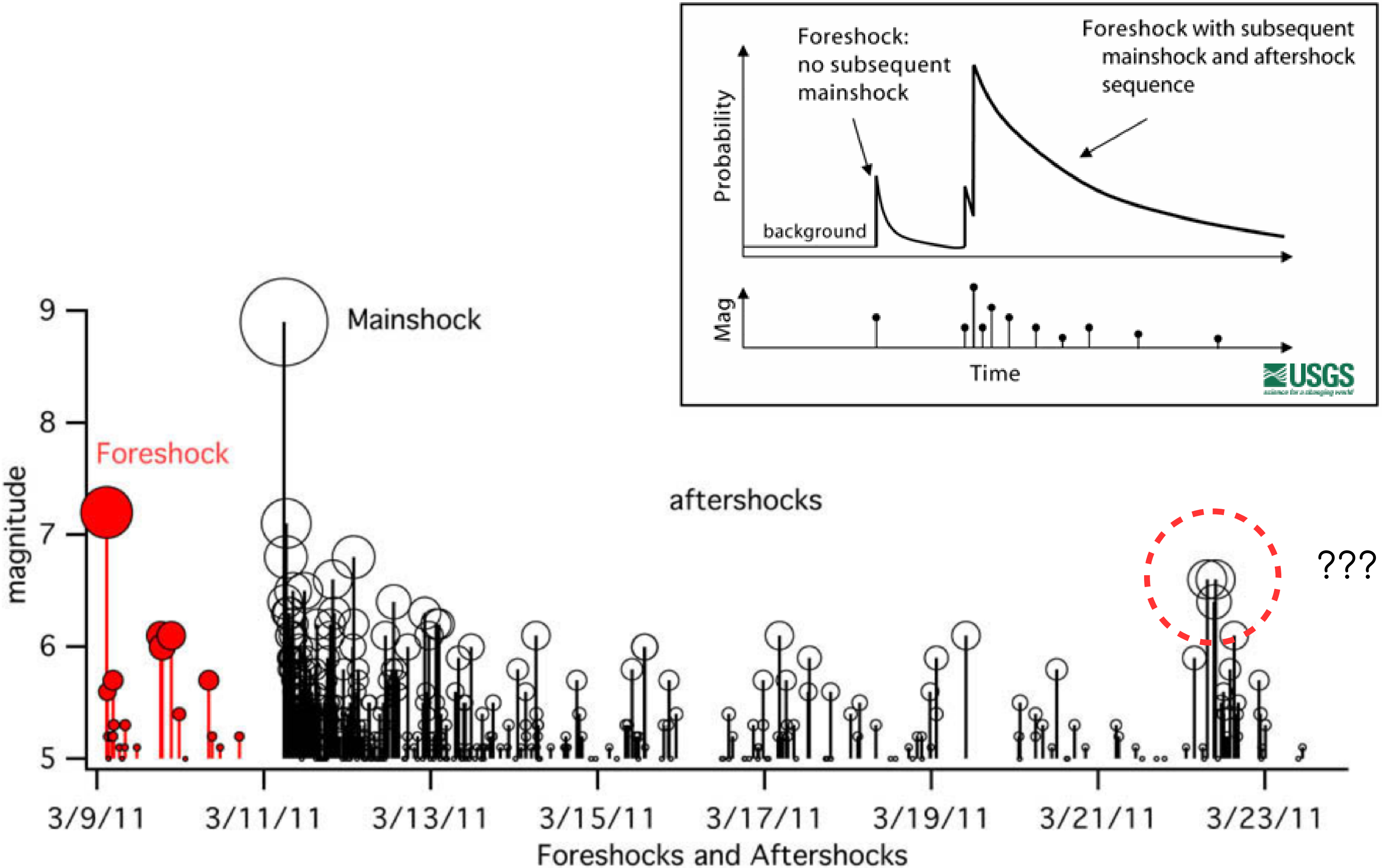
- **Main-shocks** are independent earthquakes caused by the tectonic loading, or in the case of seismic warms by stress transients that are not caused by previous earthquakes

- **Aftershocks and foreshocks** corresponds to earthquakes triggered by static or dynamic stress changes, seismically-activated fluid flows, after-slip, etc., hence by mechanical processes that are at least partly controlled by previous earthquakes.

Foreshokes and Afershocks



Foreshocks and Aftershocks



Omori's Law

Aftershock rate decay, R is described by Omori's law (Omori, 1894; Utsu, 1961):

$$R(t) = \frac{K}{(t+c)^p}$$

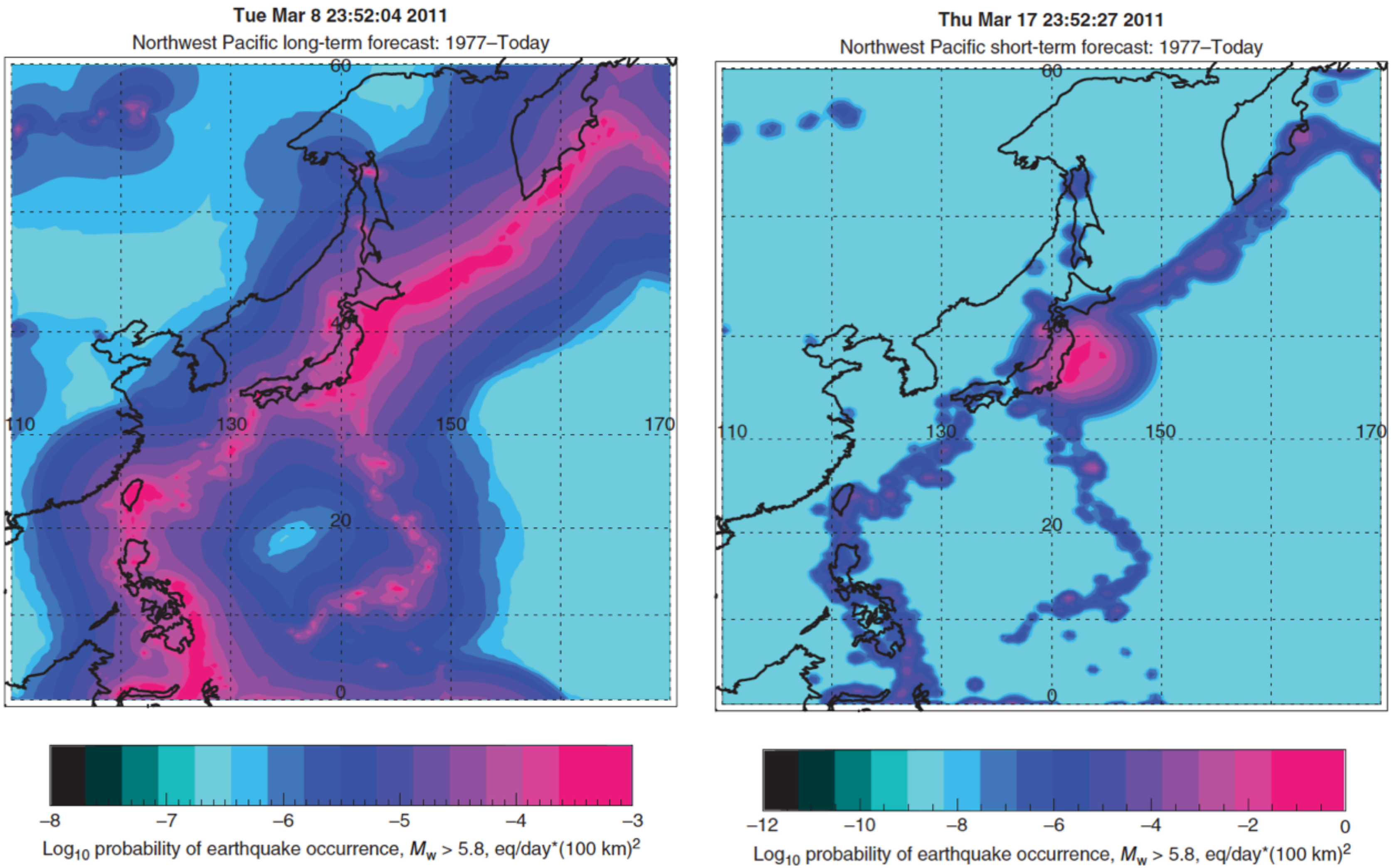
where K is the productivity for each earthquake, c and p are empirical constants and t is the time since triggering shock occurrence.

Fluctuations of p values exist for each aftershock sequence. For tectonic seismicity, p values are usually found in the 0.8–1.2 range.

NOTE: Aftershock sequences also typically follow the Gutenberg-Richter law of size scaling.

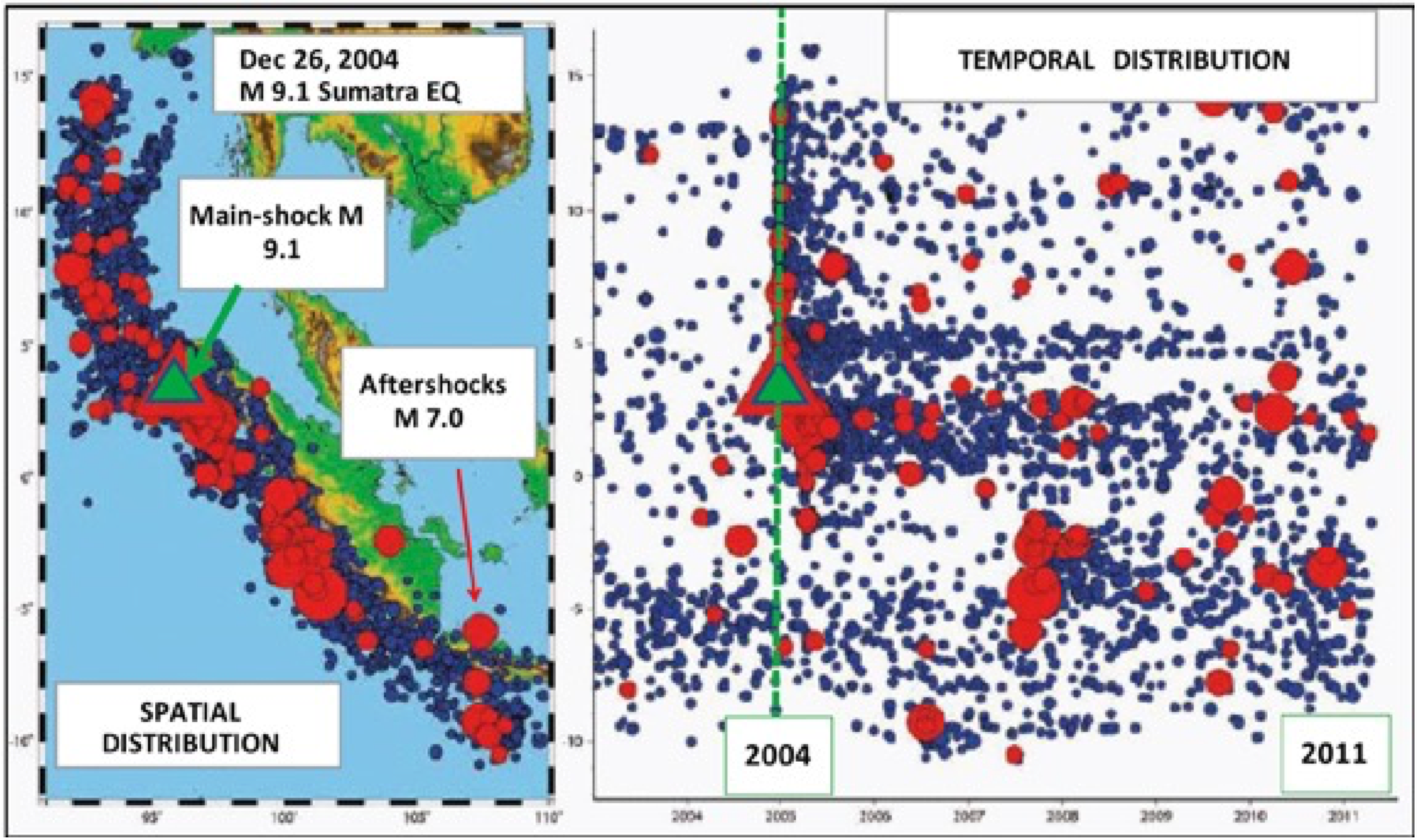
L-T vs. S-T Rate Forecast

Tohoku Earthquake (Mw=9.2, 2011)



Spatial-Temporal Distribution

Aftershock of Sumatra 2004 Earthquake



Declustering Methods

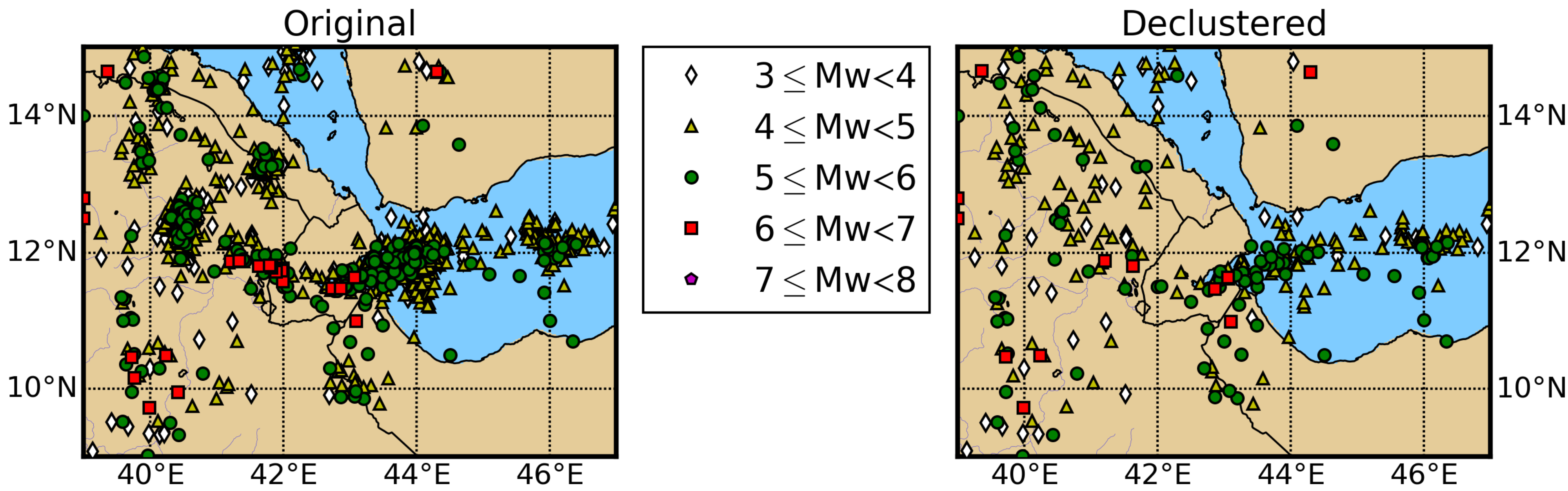
Many algorithms exist, however the most famous (and simpler) are the window (space-time domain) based methods:

- Gardner & Knopoff, 1972
- Gardner & Knopoff, 1974
- Uhrhammer, R. (1986)
- Gruenthal, G (1985)

Different S-T windows!

For each earthquake with magnitude M, foreshocks and aftershocks are identified if they occur within a specified time interval (T), and within a distance interval (S) from the main event. Space and time windows are scaled according to the size of the main shock.

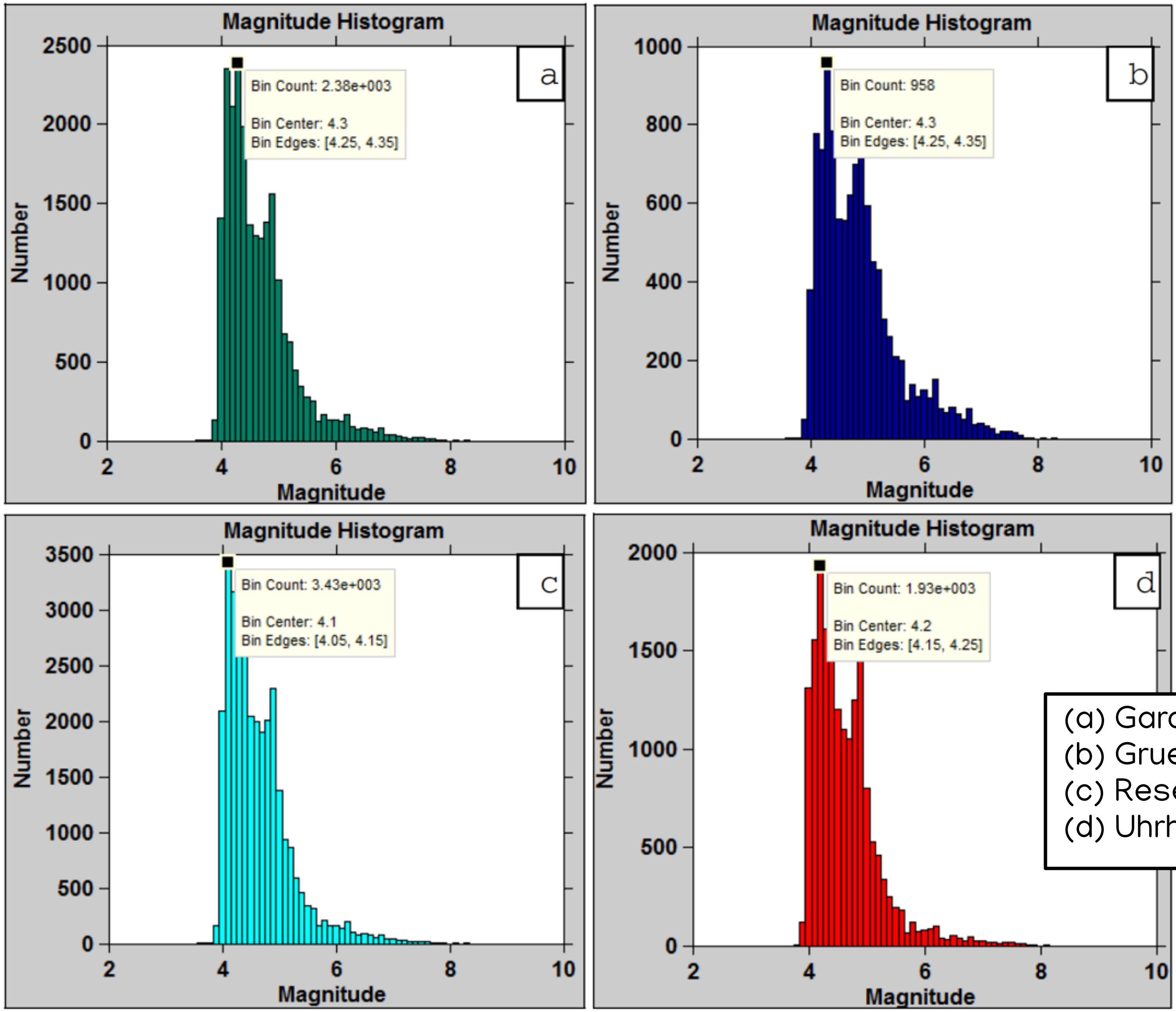
Example



Unfortunately, these algorithms do not distinguish between direct and indirect aftershocks, i.e., 1st-generation aftershocks and aftershocks of aftershocks.

Removing too many events can bias occurrence estimates and the related hazard results.

Comparing Results



(a) Gardner and Knopoff (1974)
(b) Gruenthal (1985),
(c) Reseanberg (1985)
(d) Uhrhammer (1986).

Good Practice: Try to verify declustered catalogue is Poissonian!

Other Advances Methods…

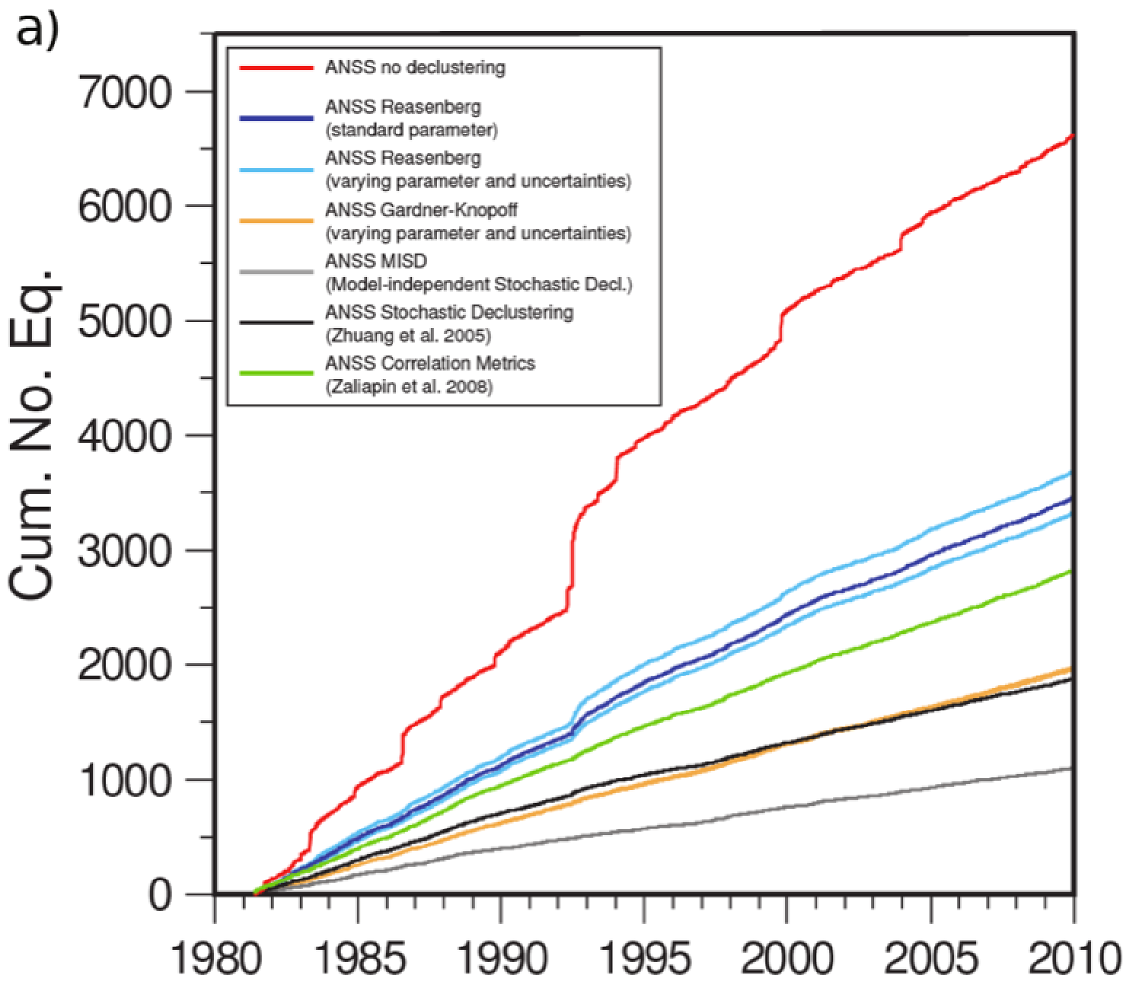
1) Linked window: every event has a window.
Clusters are maximal sets of events such that each is in the window of some other event in the group.
Replace cluster by single event: first, largest, "equivalent"

2) Stochastic Methods:
Zhuang et al. (2002) use an "epidemic-type aftershock sequence" (ETAS) model and maximum likelihood to estimate contributions to the total seismicity from the background rate and branching structure.

3) Non-Parametric Methods:
Hainzl et al. use the distribution of inter-event times to derive a nonparametric estimate of the rate of mainshocks.

4) Model-Independent Stochastic Declustering
5) Single-link cluster analysis
6) Declustering methods based on correlation metric

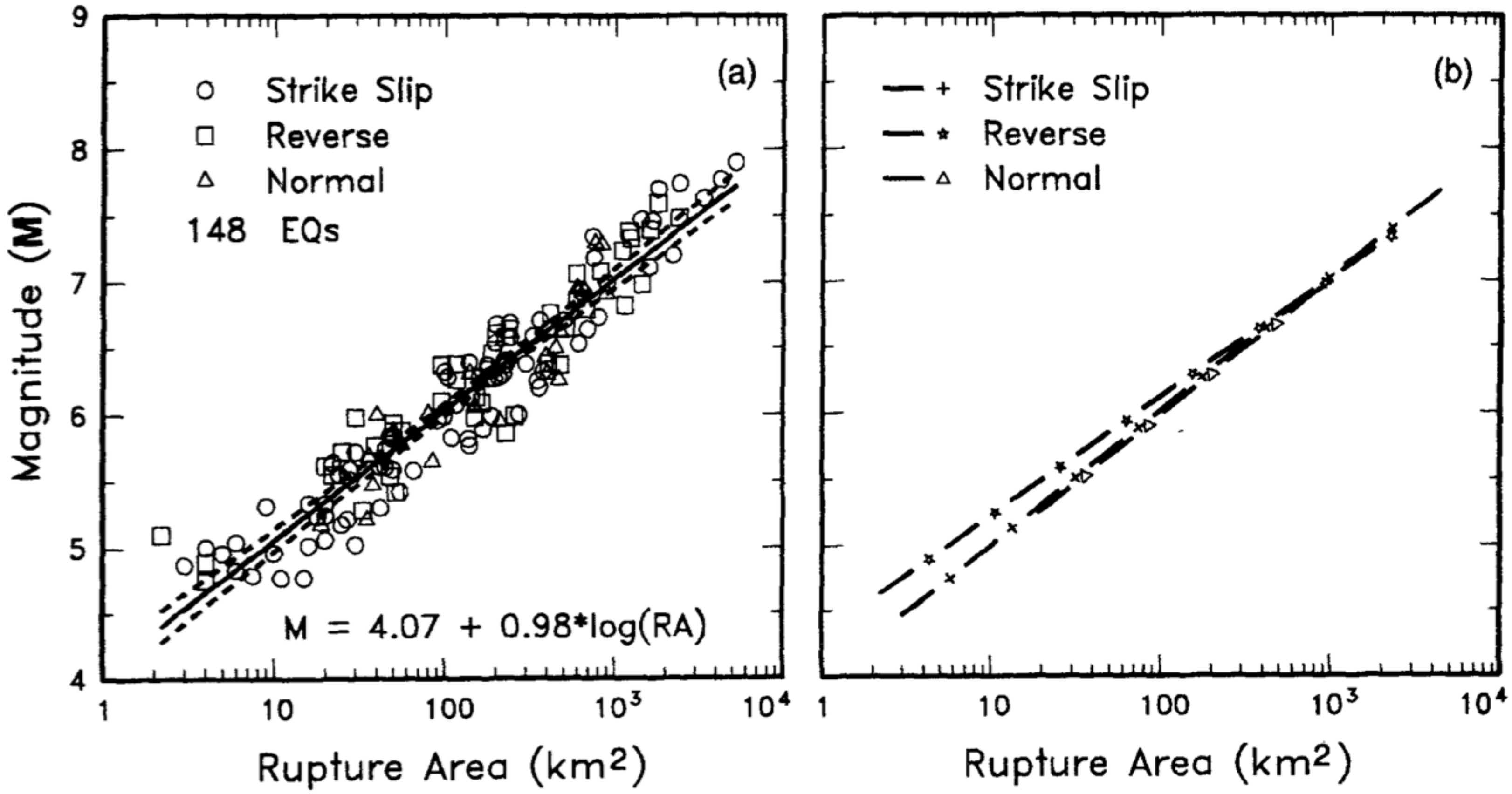
Other Advanced Methods



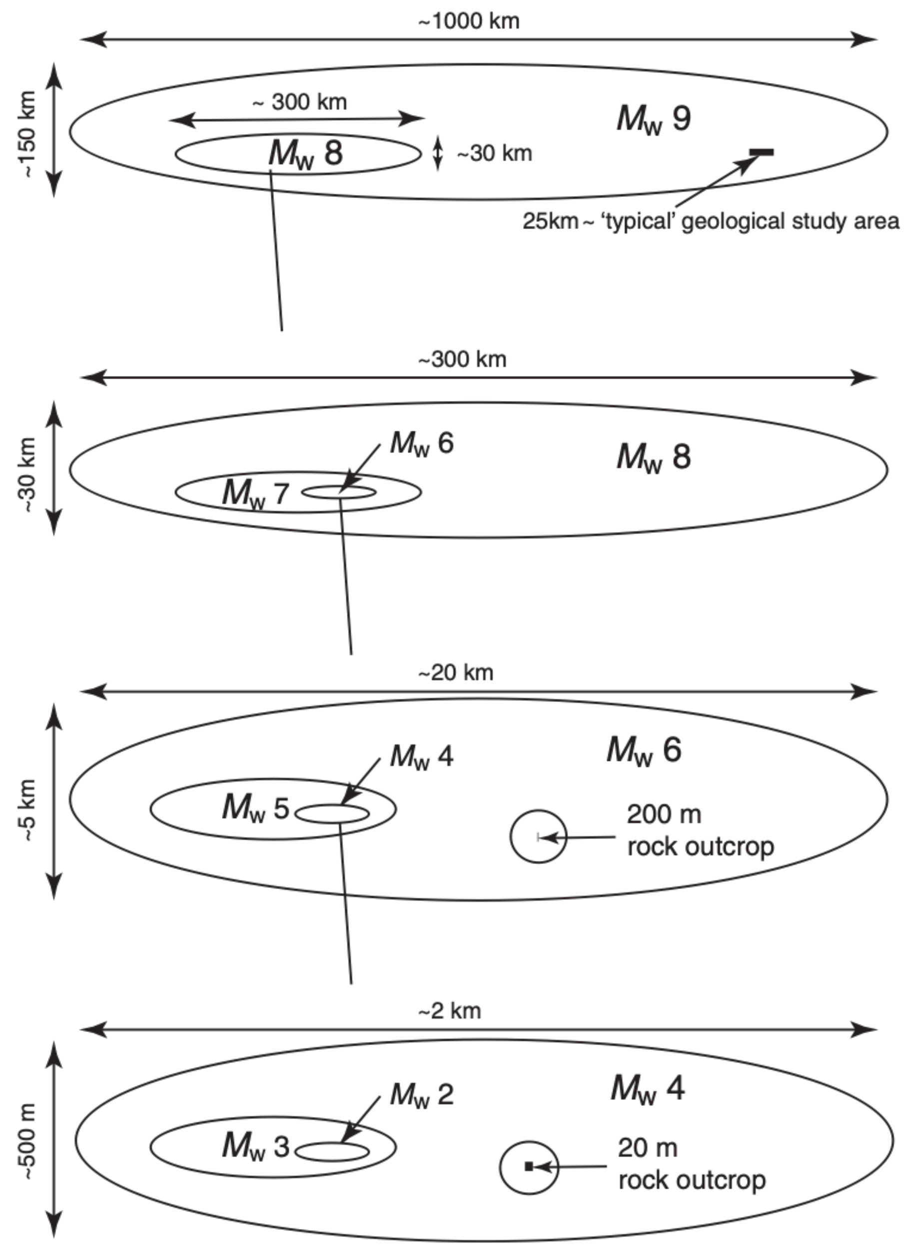
Magnitude-Scaling Relations

Magnitude-area scaling relationship proposed by Wells and Coppersmith (1994)

log(A) = −3.49+0.91M   Sigma=0.24



Magnitude–Scaling Relations



Occurrence from Geodesy

The variation of the seismic moment over time (moment rate) can be written as function average slip derivative on the fault, also called slip rate:

$$\bar{M}_0 = \mu\, A\, \dot{D}$$

Moreover, we know that:

$$M_w = \frac{2}{3}\log\left(M_0\right) - 10.7$$

Therefore, knowing the slip rate of a fault could potentially provide information on the occurrence of events of a certain magnitude.

Estimating Slip Rates

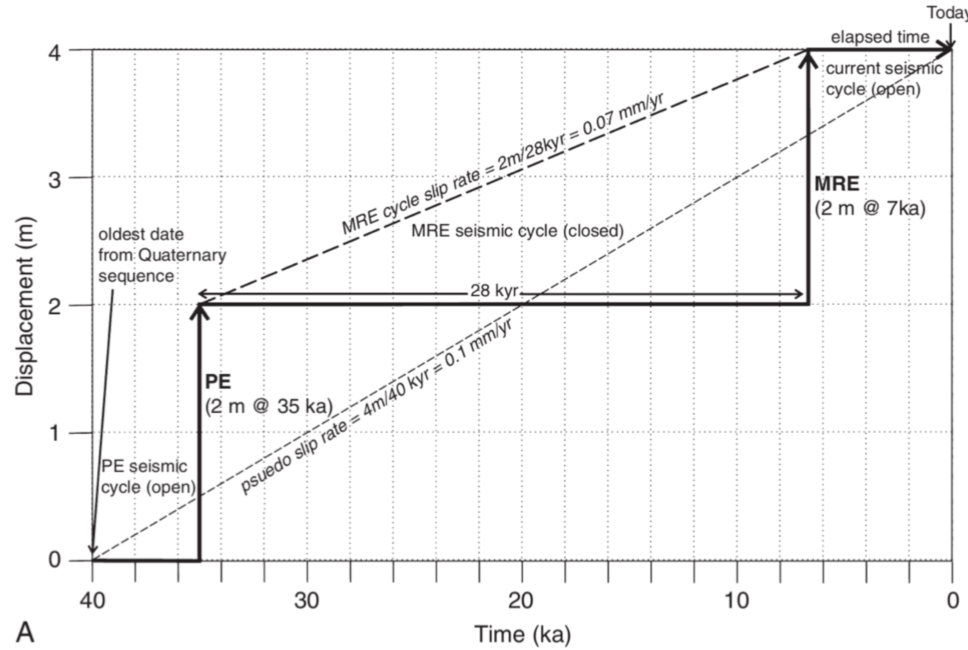
Slip rate estimates from faults can be obtained in basically two ways:

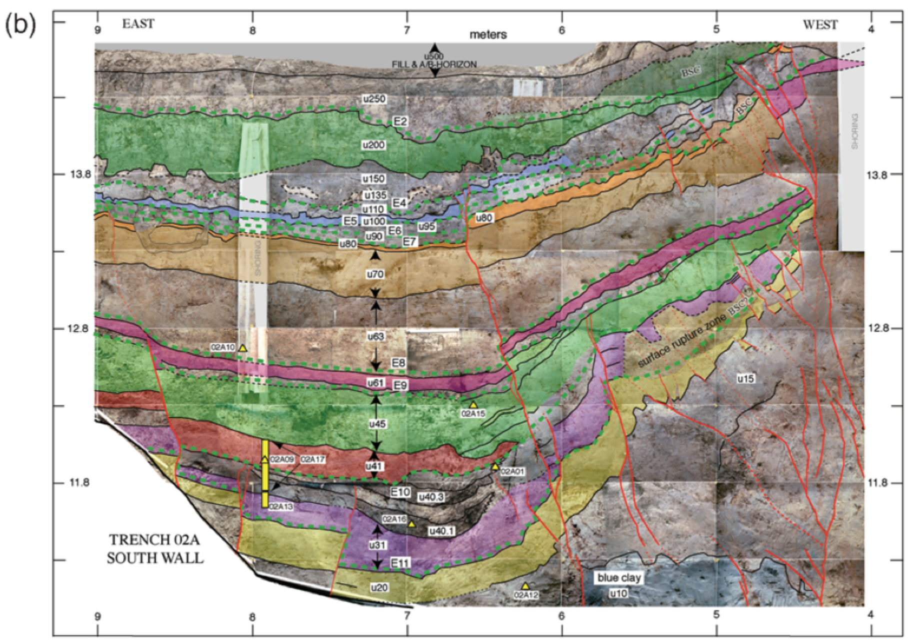
(1) from direct investigation of exposed faults (e.g. geochronological and paleoseismological analysis)
(2) from geodetic observations (e.g. GPS)

Such estimates, however, could be rather imprecise or questionable, as they rely on the assumption that slip rate:

a) is rather constant over long periods (necessary for 1)
b) can be projected back to the past (necessary for 2).

Slip Rate from Geochronology

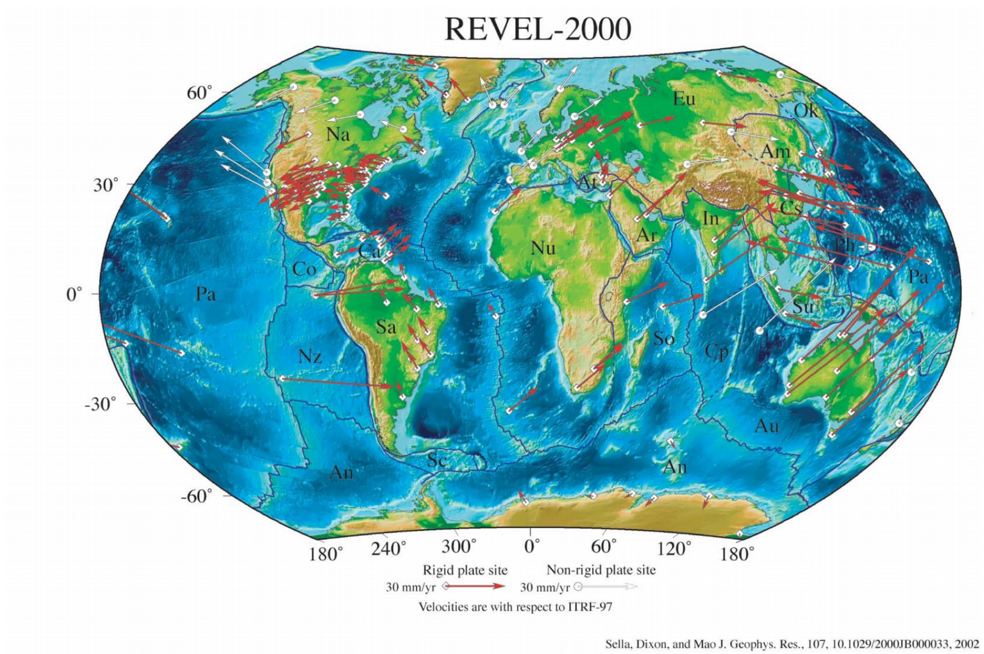


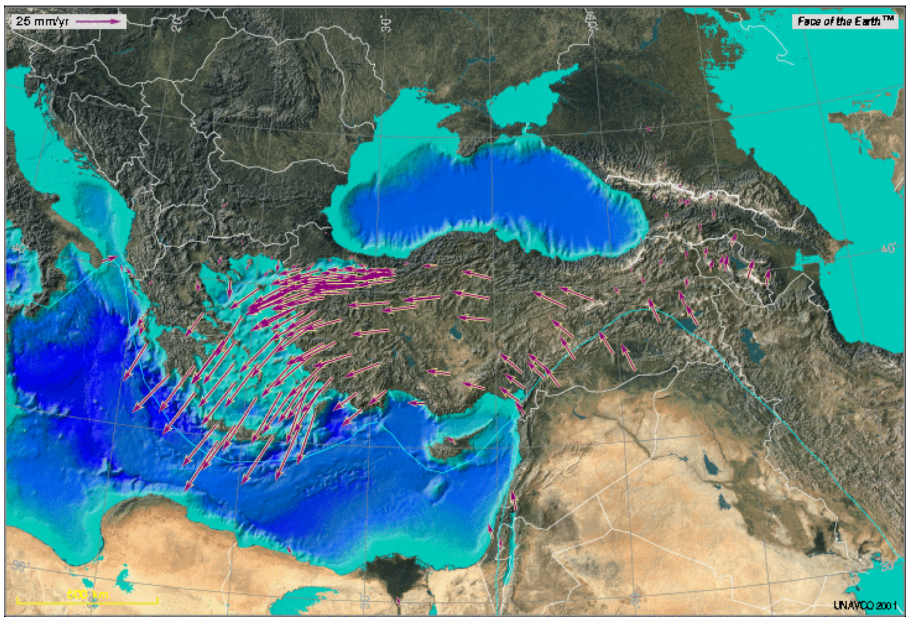


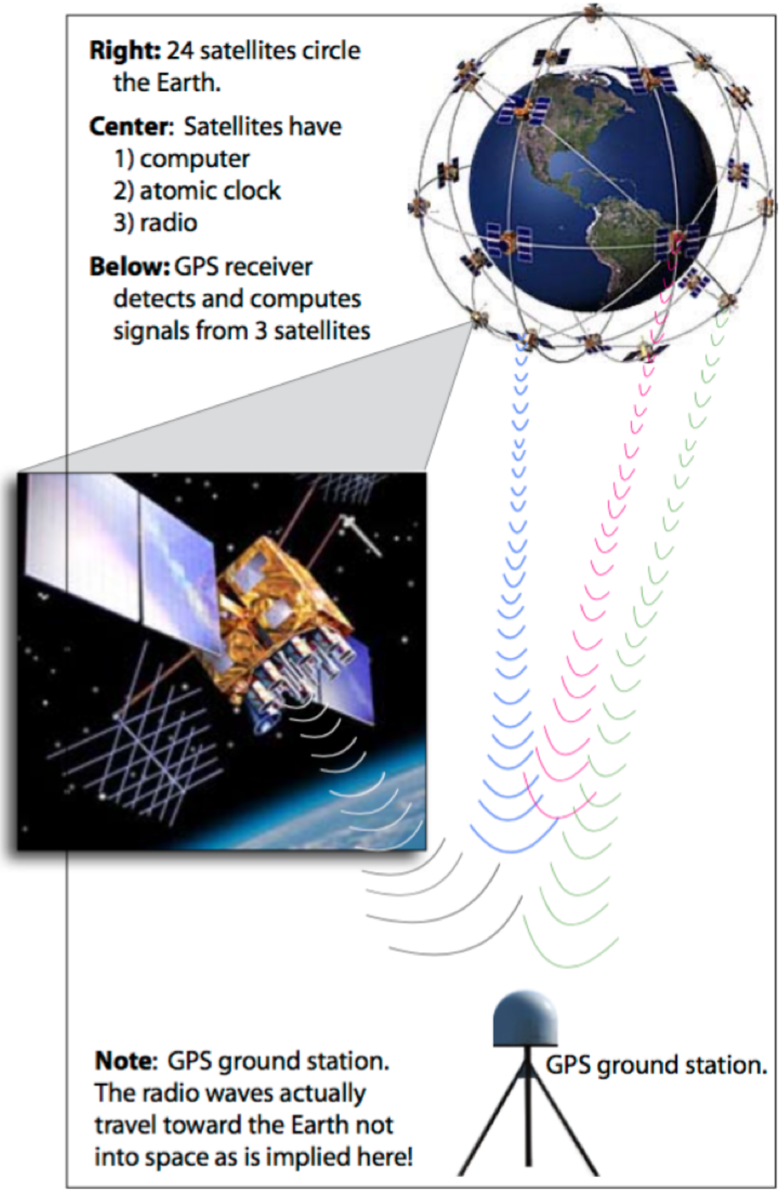
Dates of the layers can be estimated from radiocarbon dating (to about 4000 years)

For areas where faults are buried below the surface (or slowly deforming regions) paleoliquefaction features can also provide a chronology

Slip Rate from GPS







Occurrence From Slip Rates

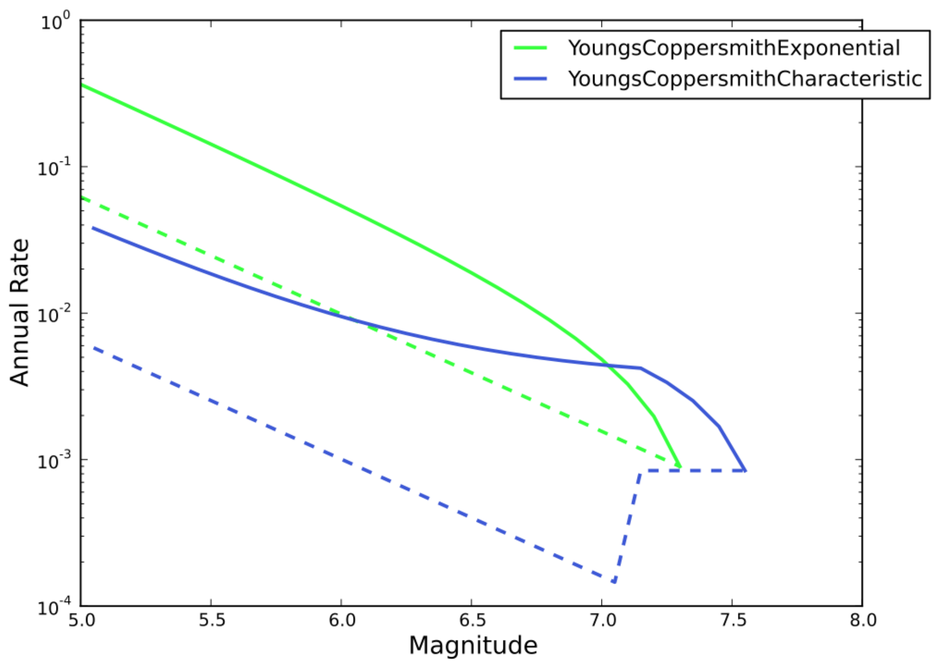
Different models exist to model occurrences from slip rate, such as:

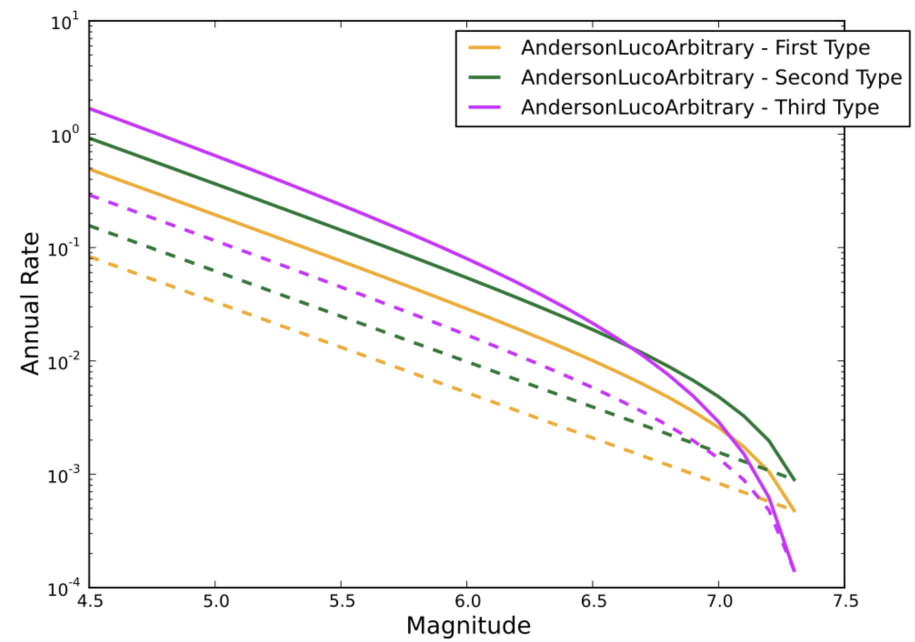
1) Simplified characteristic
2) Anderson & Luco (1983)
3) Youngs & Coppersmith (1985)

$$N(M_{MIN}) = \frac{c\mu A\bar{s}}{\int_{M_{MIN}}^{M_{MAX}} 10^{(1.5M+9.05)} f_M(m)\,dm}$$

$$N(M_W \geq M) = \frac{\bar{d}-\beta}{\bar{d}}\left(\frac{\mu A\dot{s}}{M_O(M_{MAX})}\right)e^{\beta(M_{MAX}-M)}$$

$$N(M_W \geq M) = \frac{\mu A\dot{s}(1.5-b)\left(1-e^{-\beta(M_{MAX}-M)}\right)}{bM_O(M_{MAX})e^{(-\beta(M_{MAX}-M))}}$$





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